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# A THEORETICAL MODEL OF THE ELECTROMAGNETIC EMISSION EFFECT OF ROCK MEMORY

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The electromagnetic emission memory effect (EEME) is observed under cyclic mechanical loading of rock specimens with an increase in the load amplitude from cycle to cycle. The EEME is a jump-type increase in the activity of electromagnetic emission (EME) of a specimen after the largest value of load in the previous cycle is reached [1, 2]. By the activity is meant the number of EME pulses per unit time.

Active experimental EEME studies have called for a theoretical understanding of the mechanism of this phenomenon, and thus a theoretical model of memory effects [3] has been designed. Based on linear fracture mechanics, this model [3] explains the peculiarities of some rock-memory effects in terms of stresses. However, it was designed for a low level of loads (below the long-time strength limit) and does not incorporate the mechanism of electromagnetic pulse generation.

In this paper, we extend the model of [3] to the entire range of loads from zero to compressive strength. Main attention is given to the explanation of the experimentally observed EEME regularities.

Before loading, the rock is assumed to contain $S$-shaped microcracks (Fig. 1), whose existence has been confirmed in experiments on model materials and rock specimens [4]. An $S$-shaped microcrack is a combination of a shear crack ($BC$ in Fig. 1) and two tensile cracks ($AB$ and $CD$). The latter branched off from the shear crack under natural conditions of rock loading under the action of shear in the plane $BC$ [3–5]. In the perpendicular direction to the figure's plane, the crack length is assumed to be equal to unity, because we consider a two-dimensional problem. The crack sides are not in contact. The distance between $S$-shaped cracks before laboratory loading is much greater than the crack dimensions.

Under natural conditions, the rock was subjected to complex loading with repeated variations in both the values of and directions of action of the principal stresses. The initial lengths of tensile cracks $l_i^0$ ($i$ is the crack number) are therefore random. Let us assume that all shear cracks have the same length $L$ but are distributed randomly (the angle $\alpha_i$ is a random quantity). This corresponds to chaotic rock fracturing.

When such a material is loaded by a uniaxial load $\sigma$ (Fig. 1), the stress state at the mouth of a tensile crack is characterized by two stress intensity factors (SIF): $K_{\rm I}$ and $K_{\rm II}$.

The value of $K_{\rm I}$ is determined by the action of tensile forces at points $B$ and $C$ (Fig. 1) due to the shear in the plane $BC$. The $K_{\rm I}$ SIF of an $S$-shaped crack is determined as [3, 5]

$$K_{\rm I} = \frac{\sigma L \sin 2\alpha_i}{4\zeta\sqrt{l_i}} \tag{1}$$

($\zeta$ is a dimensionless coefficient [3]) and $K_{\rm II}$ is determined by shear in the tensile-crack's plane

$$K_{\rm II} = \frac{\sigma}{2}\xi\sqrt{\pi l_i}\sin 2\alpha_i. \tag{2}$$

Here $\xi$ is a dimensionless coefficient that characterizes the configuration of the $S$-shaped crack (the effect of the free plane $BC$).

The $K_{\rm II}$ value is small for small $l_i$, and the crack grows for

$$K_{\rm I} \geqslant K_c \tag{3}$$

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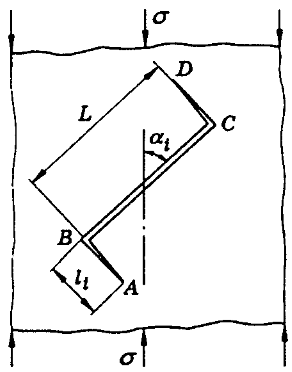


Fig. 1

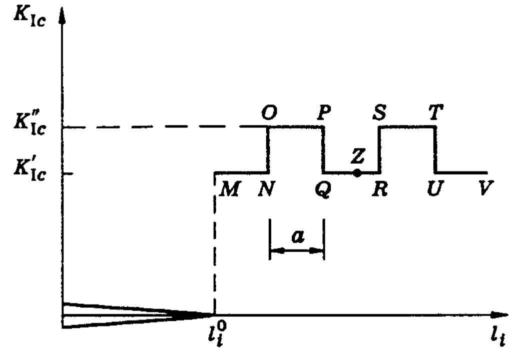


Fig. 2

($K_{Ic}$ is the critical SIF for normal-separation cracks).

An increase in the length of tensile cracks during their growth causes a decrease in $K_I$. As a result, the cracks propagate under large loads if

$$K_{II} \geqslant K_{IIc} \tag{4}$$

($K_{IIc}$ is the critical SIF for a longitudinal-shear crack).

Rocks are media that are not homogeneous, at a microlevel, in their strength parameters $K_{Ic}$ and $K_{IIc}$. This inhomogeneity is primarily caused by the rock-grain size, because each grain and each intergranular contact display their own strength.

Since it is impossible and unreasonable to consider the actual character of the medium's inhomogeneity, we assume that the path of tensile-crack propagation (rays $BA$ and $CD$ in Fig. 1) contains regions of two $K_{Ic}$ values: $K'_{Ic}$ and $K''_{Ic}$ (Fig. 2). The lengths of all constant-strength regions are equal to $a$.

As the load increases from zero to some value $\sigma$, $K_I$ increases monotonically, and, for $\sigma = 4\zeta K'_{Ic}\sqrt{l_i^0}/(L\sin 2\alpha_i)$, the $i$th tensile crack begins to grow. Since $l_i^0$ and $\alpha_i$ are random quantities, the cracks will start to grow at different times. An increase in load will give rise to an increasing number of cracks.

Crack propagation subject to (3) will be stable, because, as follows from (1), the $K_I$ value decreases with an increase in $l_i$, and the crack will grow only with increasing $\sigma$. In the constant-strength region ($K_{Ic} = K'_{Ic} = \text{const}$) the crack grows without stopping as the load increases.

When the crack approaches the $OP$ region (Fig. 2), it stops because the given level of load does not allow it to overcome the $NO$ barrier. With an increase in stress to $\sigma = 4\zeta K''_{Ic}\sqrt{l_i^0 + a}/(L\sin 2\alpha_i)$, crack propagation resumes and will continue over the entire $OP$ region with increasing $\sigma$.

At point $P$, $K_{Ic}$ decreases to $K'_{Ic}$. In this case, the $\sigma$ value already exceeds that required for crack propagation, and the crack grows stepwise to a length that corresponds to the attained level of load. Ideally, the growth occurs instantaneously for $\sigma = \text{const}$.

Further development of the crack can follow in two ways. When the $K''_{Ic}$ value is large enough compared with the value of $K'_{Ic}$

$$K''_{Ic} \geqslant K'_{Ic}\sqrt{(l_i^0 + 3a)/(l_i^0 + 2a)}, \tag{5}$$

the crack can grow over the entire $QR$ region for $\sigma = \text{const}$. Near the $RS$ barrier at which crack propagation stops, the load must be increased again.

When $K''_{Ic}$ is not much larger than $K'_{Ic}$ [condition (5) is not satisfied], starting with a certain point $Z$ crack propagation will be controlled by the parameter $K'_{Ic}$ (Fig. 2). In approaching the $RS$ barrier, the crack first stops and then resumes its propagation as soon as the condition $K_{Ic} \geqslant K''_{Ic}$ is satisfied.

The two indicated possibilities are illustrated in Fig. 3, where the dashed curves show crack length versus load in a $K_{Ic}$-homogeneous material (curve 1 corresponds to $K_{Ic} = K'_{Ic}$, and curve 2 to $K_{Ic} = K''_{Ic}$).

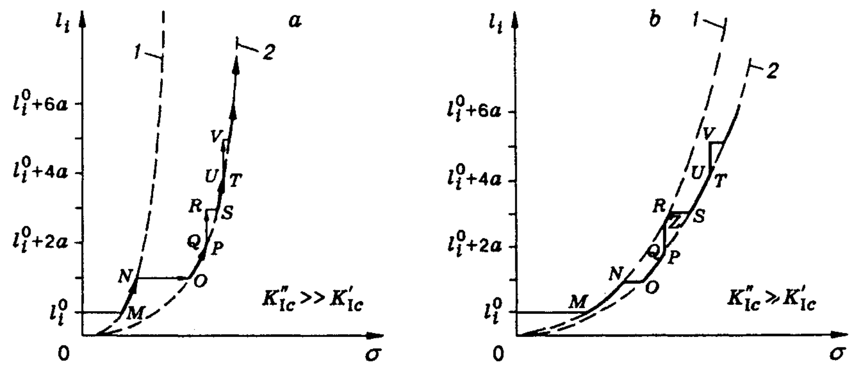


Fig. 3

Figure 3a shows that crack propagation in a heterogeneous medium with strongly different $K'_{Ic}$ and $K''_{Ic}$ values is actually controlled only by the parameter $K''_{Ic}$, i.e., the largest from the critical SIF. At the same time, Fig. 3b and formula (5) demonstrate that in a medium with close $K'_{Ic}$ and $K''_{Ic}$, crack propagation will also be controlled, sooner or later, only by one parameter ($K''_{Ic}$ SIF). We shall therefore consider a medium with greatly different $K'_{Ic}$ and $K''_{Ic}$ values (Fig. 3a).

The crack sides are electrified upon crack growth due to the separation of opposite charges on the fracture surfaces formed. While opening the crack, the potential between the crack sides increases, and when the critical voltage is reached, the discharge appears [6]. The discharge current decreases according to an exponential law and, as a result, the emission pulses acquire a saw-tooth shape, which was observed experimentally:

$$E = E_0 \exp(-t/\tau).$$

In this case, $E$ is the electric-field voltage in the EME pulse; $E_0$ is the initial $E$ value; and $\tau$ is a discharge-time constant.

Thus, to crack propagation on $2a$ (from stop to stop) corresponds one saw-tooth EME pulse. Here by the stops are meant the $RS$-type regions in Figs. 2 and 3.

Let us determine the dependence of the emission activity $\dot{N}$ (the number of pulses per second) on the stress $\sigma$. Figure 3a shows that

$$\dot{N} = \frac{d[(l_i - l_i^0)/2a]}{d\sigma}\frac{d\sigma}{dt}.$$

Substituting

$$l_i = (\sigma L \sin 2\alpha_i / 4\zeta K''_{Ic})^2, \tag{6}$$

we obtain

$$\dot{N} = \frac{\sigma L \sin 2\alpha_i}{4\zeta a K''_{Ic}}\frac{d\sigma}{dt}. \tag{7}$$

As is seen from (7), the EME intensity increases with an increase in the load, which was confirmed in numerous experiments [7].

The following fact, which was established experimentally, can be explained within the framework of the suggested model as well. The total number of pulses before fracture $n_\Sigma$ decreases in transition from hard to soft rocks and also, within the same rock, in transition from more hard to less hard varieties [8]. As a matter of fact, the $K''_{Ic}$ values for the hardest grains are equal for rocks of the same mineralogical composition and are close for rocks of various species. The total number of pulses that are accumulated with increasing load from 0 to $\sigma$ has the form

$$n_\Sigma = \frac{l_i - l_i^0}{2a} \simeq \frac{(\sigma L \sin 2\alpha_i / 4\zeta K''_{\mathrm{Ic}})^2}{2a} \tag{8}$$

(in the latter transition we have allowed for the fact that as the load increases, the condition $l_i \gg l_i^0$ holds). Formula (8) shows that for equal $L$, $\zeta$, $K''_{\mathrm{Ic}}$, and $a$, the rocks that can sustain larger stresses without fracture emit a larger total number of EME pulses.

Another experimental fact is the increase in the amplitude of EME pulses with increasing $\sigma$. Figure 3a shows that for crack propagation on $2a$, a progressively smaller increase in $\sigma$ is needed as the load increases. By virtue of the proportionality of the load and time, the growth of a crack on $2a$ takes less time. When the crack sides are separated, before the discharge appears the surface charge is neutralized by the current passing through the rock. Since the separation takes less time, the charge leakage decreases, thus giving rise to an increase in the initial value of the discharge current and, hence, in $E_0$.

We now explain the EEME regularities using the suggested model. Let in the first cycle the specimen be loaded to a maximum stress $\sigma_m$. This means that after unloading the crack length obeys equality (6), where $\sigma$ must be replaced by $\sigma_m$.

Upon loading from 0 to $\sigma_m$ in the second cycle, condition (3) is not satisfied, and the cracks do not grow. With the load level $\sigma = \sigma_m$ attained, the cracks begin to grow once again, and the EME-activity value is reached.

Let us consider now the processes of crack growth when the compressive strength is approached. In this case, the tensile crack lengthens. As a result, crack propagation is controlled by condition (4) and is unstable in this case. Starting once, the process cannot stop, because $K_{\mathrm{II}} \sim \sqrt{l_i}$. The growth will thus continue until either the crack enters a free surface (the specimen surface or the surface of another $S$-shaped crack) or the secondary tensile crack forms.

This is possible because the tensile crack now becomes a shear crack with SIF determined by (2). The secondary-crack growth will occur for $\sigma = \text{const}$, because the attained load is sufficient for the propagation of even larger cracks.

The coalescence and continuous growth of shear and tensile cracks lead to violation of the agreement between the load values and crack lengths. This, in turn, causes the disappearance of EEME.

Thus, the suggested model explains the experimentally observed EEME regularities of brittle rocks.

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